THE COSMIC MICROWAVE BACKGROUND DIPOLE AS A COSMOLOGICAL EFFECT

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ABSTRACT

A conventional explanation of the dipole anisotropy of the cosmic microwave background (CMB) radiation is in terms of the Doppler effect: our galaxy is moving with respect to CMB frame with $\sim 600\ km\ s^{-1}$. However, as the deep redshift surveys fail to reveal a convergence of the large scale flow to zero at distances as large as $d \sim H^{-1} 15,000\ km\ s^{-1}$ (Lauer & Postman, 1994), the uniqueness of the conventional interpretation has to be investigated. A possible alternative might be a cosmological entropy gradient, as suggested by Paczyński & Piran (1990). We find that contrary to that suggestion a quadrupole anisotropy is generically of the same order of magnitude as the dipole anisotropy (or larger) not only for adiabatic but also for iso-curvature initial perturbations. Hence, the observed dipole cannot be explained with a very large scale perturbation which was initially iso-curvature.

Subject headings: cosmic background radiation - cosmology - gravitation

1. INTRODUCTION

The dipole moment of the CMB is usually interpreted to be the result of a Doppler effect caused by our motion with respect to CMB frame (cf. Kogut *et al.* 1993 and references therein). According to this interpretation the CMB, as well as galaxies (when averaged over a large enough volume) define the local standard of rest. Our galaxy, together with its neighbours moves with respect to the local standard of rest. When the velocities of galaxies in a large enough volume are measured they should be found to be at rest, naturally after the allowance is made for the overall Hubble expansion.

The observations available so far fail to provide a clear support for this picture. The recent most troublesome result is that of Lauer & Postman (1994) who find that the frame defined with the 119 Abell clusters of galaxies within $15,000\ km\ s^{-1}$ is moving at $\sim 700\ km\ s^{-1}$ with respect to the local standard of rest as defined by the CMB. This scale is so large that it is difficult to accomodate within most currently available models of the formation of large scale structure in the universe (Strauss et al. 1994). This trouble persisted for many years (cf. Paczyński & Piran 1990, hereafter PP, and references therein) and it justifies a search for alternative interpretations of the CMB dipole anisotropy. PP proposed that entropy gradient on a scale larger than the current horizon could give rise to a dipole moment while keeping the quadrupole unmeasureably small.

The purpose of this paper is to demonstrate that the PP proposal was incorrect. PP used Tolman-Bondi cosmological model with no pressure to demonstrate that the very long wavelength density perturbations give rise mostly to a quadrupole anisotropy, while entropy perturbations show up as a dipole. The reason for ignoring pressure was the currently very small value of density and pressure due to CMB. The CMB was dynamically important only at redshifts larger than $z_{eq} \sim 10^4$, and it seemed safe to ignore it, or at least it seemed to be a reasonable first approximation. It turns out that was a conceptual mistake. The purpose of this paper is to demonstrate that no matter how small is the current contribution of radiation to the closure density any large scale iso-curvature (entropy) perturbation generically gives rise to a quadrupole CMB anisotropy which is larger than the dipole anisotropy. We demonstrate this in section 2 for a plane wave, and in section 3 for a modified Tolman-Bondi model. Finally, a simple qualitative demonstration of the fact that the initially iso-curvature (entropy) perturbation generically gives rise to a density perturbation (cf. Peebles, 1993, hereafter P93) is presented in section 4, together with a discussion of other possibilities.

2. PLANE WAVE PERTURBATIONS IN A FLAT UNIVERSE

In the synchronous gauge (P93) the perturbed metric of the flat universe model has the form:

$$ds^2 = c^2 dt^2 - a^2(t)(\delta_{jm} - h_{jm})dx^j dx^m \tag{2.1}$$

where t is the cosmic time, $a(t)$ the scale factor in the unperturbed model, δ_{jm} is the Kronecker delta symbol and $h_{jm}(t, x^l)$ are the small perturbations to the metric. The spatial coordinates are Cartesian, Latin indices go through 1,2,3. We are interested in the growing, scalar modes of perturbations to the metric (Lifschitz, 1946). In the case of the plane wave perturbation with the wave vector $\mathbf{k}$ along the x^3 axis, the only nonvanishing components of h_{jm} are the diagonal terms and $h_{11} = h_{22}$ because of the symmetry. Thus there are two independent variables defining tensor h_{jm} and we choose $h = h_{11} + h_{22} + h_{33}$ and $H = h_{11} - h_{33}$ for the purpose.

In our model the matter consists of two independent components, which interact which each other by gravity only. The first component is non relativistic (NR) matter (for example baryons) and it is characterised by the present dimensionless density $\Omega_{NR} = \rho_{NR}/\rho_c$, where $\rho_c \equiv 3H_0^2/8\pi G$ is the critical cosmological density, H_0 is the Hubble's constant and G is the constant of gravity. The second component is ultra-relativistic (UR) matter (for example electromagnetic radiation) and it is characterised by the present dimensionless density $\Omega_{UR} = \rho_{UR}/\rho_c$. Since we are using the flat unperturbed cosmological model, we add the cosmological constant to be in agreement with unperturbed evolutionary equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}(\epsilon_{NR} + \epsilon_{UR}) + \frac{1}{3}\Lambda c^2 \tag{2.2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon_{NR} + \epsilon_{UR} + 3P_{UR}) + \frac{1}{3}\Lambda c^2 \tag{2.3}$$

where dots mean the time differentiation, $\epsilon_i \equiv \rho_i c^2$ depict the full energy densities of various components and P_i - their pressures. We introduce dimensionless cosmological constant $\lambda = \frac{1}{3}\Lambda c^2/H_0^2$. In a flat model we have:

$$\Omega_{NR} + \Omega_{UR} + \lambda = 1 \tag{2.4}$$

In some cases λ may be negative.

Throughout this paper we assume that all perturbations of all non-relativistic matter follow strictly the perturbations of baryons. We also assume that all perturbations of all ultra-relativistic matter follow strictly the perturbations of electromagnetic radiation. The ultra-relativistic and non-relativistic components are coupled to each other prior to

recombination, and they become decoupled when the universe becomes transparent to radiation at $z_{rec} \gg 1$.

The energy density fluctuations of different components are described by:

$$\frac{\delta\rho_{NR}}{\rho_{NR}} = \delta_{NR} \qquad \frac{\delta\rho_{UR}}{\rho_{UR}} = \frac{4}{3}\delta_{UR} \tag{2.5}$$

where δ_{NR}, δ_{UR} are the relative perturbations in particle density of the two componets. The perturbations in specific entropy $\delta_S \equiv \delta S/S$, where S is the entropy of ultrarelativistic fluid per particle of nonrelativistic fluid can be exppressed as $\delta_S = \delta_{UR} - \delta_{NR}$. (We neglect the entropy of the nonrelativistic fluid). The velocity perturbation of each fluid has the form $u^\alpha = (1/c, v^j/ac)$ where $\mathbf{v}$ is the physical velocity measured by the synchronous observers . In the scalar mode of perturbations only the correction to the expansion, Θ, enters the equations:

$$u^\alpha_{;\alpha} = 3\frac{\dot{a}}{a} + \Theta \qquad \Theta = \frac{\dot{a}}{a}\chi \tag{2.6}$$

The dimensionless variable χ is more convenient than Θ (Press & Vishniac, 1980, hereafter PV80) and we need χ_{NR} and χ_{UR} to characterize both components.

Our set of equations describing the evolution of small perturbations to the metric and fluid variables is based on P93 and PV80. We use a new time coordinate (PV80) $\eta \equiv ln(a/a_0) \equiv -ln(1+z)$, where a_0 is the present characteristic scale in the Universe and z is the redshift. The spatial gradients are already omitted in the equations, so they are valid for very large scale perturbations only. The equations are written for two fluid interacting only gravitationally (but see below):

$$h'' + \left(\frac{\ddot{a}a}{\dot{a}^2} + 1\right) h' = \frac{8\pi G a^2}{\dot{a}^2}\left(\rho_{NR}\delta_{NR} + \frac{8}{3}\rho_{UR}\delta_{UR}\right) \tag{2.7}$$

$$H'' + \left(\frac{\ddot{a}a}{\dot{a}^2} + 2\right) H' = -\frac{8\pi G a^2}{\dot{a}^2}\left(\rho_{NR}\delta_{NR} + \frac{4}{3}\rho_{UR}\delta_{UR}\right) - h' \tag{2.8}$$

$$\delta'_{NR} = (\frac{1}{2}h' - \chi_{NR}) \tag{2.9}$$

$$\delta'_{UR} = (\frac{1}{2}h' - \chi_{UR}) \tag{2.10}$$

$$\chi'_{NR} + \left(\frac{\ddot{a}a}{\dot{a}^2} + 1\right)\chi_{NR} = 0 \tag{2.11}$$

$$\chi'_{UR} + \left(\frac{\ddot{a}a}{\dot{a}^2}\right)\chi_{UR} = 0 \tag{2.12}$$

where we have neglected sound velocity in the nonrelativistic component and for the relativistic component we have already substituted 1/3 for its pressure to energy density ratio

and for its sound velocity square. The time derivatives of a can be substituted from Eqs. (2.2), (2.3).

We solve our equations for different values of the universe model parameters varying the density of non-relativistic component in the limits $0.01 \leq \Omega_{NR} \leq 1$ and the density of ultra-relativistic component is in the range $10^{-5} \leq \Omega_{UR} \leq 1$. The value of cosmological constant results from Eq. (2.4).

We always set initial conditions long before the time of recombination and long before the time when the energy density of the ultra-relativistic component is equal to the energy density of the non-relativistic component, whichever comes first. In this early time the radiation dominated plasma behaves like a relativistic fluid and the unperturbed model evolves like a model with relativistic equation of state. (Various terms in the equations have different redshift dependence. While $\rho_{UR} \sim (1+z)^4$ and $\rho_{NR} \sim (1+z)^3$ the terms including the cosmological constant do not depend on the redshift. Thus the effects of $\Lambda \neq 0$ can only show when z is small.) In the early times the coefficients in the above equation set remain constant so one may find independent modes of perturbations (Lifshitz, 1946, P93 and references therein). We consider two types of initial conditions. The first, *adiabatic* perturbation, has three possible modes in the relativistic Universe. The most natural of them , with growing rate proportional to a^2, remains regular at $t \to 0$ and has vanishing velocity ($\chi_{NR} \equiv 0$, $\chi_{UR} \equiv 0$). Another growing mode in the early epoch with amplitudes $\sim a^1$ has nonvanishing velocity and irregular some of the metric components when $t \to 0$ (P93). Since the rate of instability in this mode is slower as compared to the previous one, it can not appreciably influence the present Universe unless the initial conditions are fine tuned. The decaying mode is of no interest: first it cannot produce any significant perturbation to the present Universe, second it is unphysical (PV80, Bardeen, 1980). For *adiabatic* perturbations we have $\delta_{NR} = \delta_{UR}$ initially and this condition is preserved if velocities vanish. (This mimics the coupling between ordinary matter and radiation, if required, without explicitly puting it into equations).

The second, *isocurvature* type of perturbations (Peebles, 1987) is impossible in a single fluid model. With more than one fluid we are able to introduce a spatial dependence of the chemical composition of the matter not introducing perturbations to the energy density or to the metric. We just perturb the density of non-relativistic component, not changing the density of ultra-relativistic component. As long as the relativistic component dominates, no energy density perturbation arises and perturbations change slowly ($\delta_{NR} \approx const$, h', H', $\delta_{UR} << \delta_{NR}$). When nonrelativistic component becomes dynamically important the growing energy density perturbation results. Examining Eqs. (2.9) and (2.10) we see, that the entropy perturbation $\delta_S = \delta_{UR} - \delta_{NR}$ remains constant.

The two kinds of perturbations considered behave differently at the begining, but

become similar when the model becomes nonrelativistic. We are interested in the fluctuations in the microwave background caused by the perturbations. They arise between the surface of last scattering and the present epoch. To compare results in different models we normalize all gravitational instability calculations in such a way that the present amplitude of the density perturbations of non-relativistic component is the same and has the value δ. According to the above remarks it is not surpraising that the influence of both kinds of perturbations on the microwave background is similar.

To find the temperature of the CMB radiation in any particular direction on the sky $\mathbf{n}$, we have to follow rays back in this direction. The coordinate distance travelled by a particle moving with the velocity of light is given by

$$\tau(t) = \tau(t(\eta)) = \int^t \frac{c\ dt}{a} = \int^\eta \frac{c\ d\eta}{\dot{a}} \tag{2.13}$$

The position of a photon, which is now at $\mathbf{x}_0$, coming from the direction $\mathbf{n}$, was at the "time" η:

$$\mathbf{x}_\eta = \mathbf{x}_0 + \mathbf{n}(\tau_0 - \tau_\eta) \tag{2.14}$$

where we have substituted $\eta = 0$ for the present epoch. τ_0 measures the present coordinate distance to the horizon.

The temperature fluctuation of the CMB radiation coming to the observer at $\mathbf{x}_0$ from the direction $\mathbf{n}$ on the sky is given as (Sachs and Wolfe, 1967):

$$\left(\frac{\delta T}{T}\right)(\mathbf{x}_0, \mathbf{n}) = \left(\frac{\delta T}{T}\right)_{\rm rec} + \frac{1}{c}\left(\mathbf{v}_{\rm obs} - \mathbf{v}_{\rm rec}\right)\mathbf{n} - \frac{1}{2}\int_{\eta_{rec}}^{0} h'_{jm}(\eta, \mathbf{x}_\eta) n^j n^m \ d\eta \tag{2.15}$$

where the subscript "rec" denotes the quantities measured at the epoch of recombination, at the place from which rays come. The first term coresponds to the temperature fluctuations in the plasma at the last scattering surface. It is equal to $\frac{1}{3}\delta_{UR}$ in the place of emission. For a plane wave with a wave vector $\mathbf{k}$ we define the directional cosine $\mu = \cos\theta = \mathbf{n}\mathbf{k}/k$. In our case $\mathbf{k}$ is along the x^3 axis and phase factor along the ray changes like $exp(ikx^3 + ik\mu(\tau_0 - \tau))$. $k(\tau_0 - \tau_{rec}) \approx k\tau_0 << 1$ for superhorizon perturbations. Expanding and taking real part we have to the lowest order:

$$\left(\frac{\delta T}{T}\right)_{\rm rec} = -\frac{1}{3}\ k\tau_0\ \sin(kx^3)\ \delta_{UR}(\eta_{rec})\ P_1(\mu) \tag{2.16}$$

$P_1(\mu) = \mu$ is the Legendre polynomial of the first order and the above expression contributes to the dipole anisotropy of the CMB (this contribution we shall denote $D_{\rm rec}$). The quadrupole part ($Q_{\rm rec}$) is of still higher order being proportional to $(k\tau_0)^2$.

The second term is due to the Doppler shift caused by the difference in velocity of matter between recombination and the present epoch. Since we do not consider velocity

perturbations, $\mathbf{v}_{\rm rec}$ vanishes automatically. The observer peculiar velocity $\mathbf{v}_{\rm obs}$ may arise from the small scale perturbations to the gravitational field but is of no interest here.

The third term is the result of gravitational field acting on photons (the Sachs - Wolfe effect). With our definitions of h, H we have:

$$\frac{1}{2}\ h'_{jm}\ n^j n^m = \frac{1}{6}\ h'\ P_0(\mu) - \frac{1}{3}\ H'\ P_2(\mu) \tag{2.17}$$

where $P_n(\mu)$ are the Legendre polynomials .*

The following of rays is required in the integration of Eq.(2.16). The gravitationally induced temperature fluctuations are given as:

$$\left(\frac{\delta T}{T}\right)_{\rm SW}(\mu) = -\frac{1}{6}e^{ikx^3}\int_{\eta_{rec}}^{0}\ (h'(\eta)\ P_0(\mu) - 2H'(\eta)\ P_2(\mu))\quad e^{ik\mu(\tau_0-\tau_\eta)}\ d\eta \tag{2.18}$$

For a superhorizon perturbation the exponent under the integral can be expanded. We get the series in the small quantity $k\tau_0$ with coefficients being the products of P_0 and P_2 with powers of μ. We limit ourselves to the dipole and quadrupole terms:

$$D_{\rm SW} = -k\tau_0\ \sin(kx^3)\int_{\eta_{rec}}^{0}\ \left(\frac{1}{6}h'(\eta) - \frac{2}{15}H'(\eta)\right)\ \left(1 - \frac{\tau}{\tau_0}\right)\ d\eta \tag{2.19}$$

$$Q_{\rm SW} = -\cos(kx^3)\int_{\eta_{rec}}^{0}\ \frac{1}{3}H'(\eta)\ d\eta \tag{2.20}$$

One can define the density difference accross the horizon $\Delta = |\tau_0 \nabla\delta| \approx k\tau_0\delta$. For various contributions to anisotropy we have:

$$D_{\rm rec} = d_{\rm rec}\ \Delta \qquad D_{\rm SW} = d_{\rm SW}\ \Delta \tag{2.21}$$

and

$$Q_{\rm rec} = q_{\rm rec}\ \frac{\Delta^2}{\delta} \qquad Q_{\rm SW} = q_{\rm SW}\ \delta \tag{2.22}$$

In Fig.1 we show the ratio of the dipole to quadrupole CMB anisotropy ($D/Q \approx D_{\rm SW}/Q_{\rm SW}$ measuring it by the ratio of the density difference through the horizon to the density perturbation amplitude (Δ/δ) which is a small quantity being the ratio of the present horizon size to the present scale of perturbation. The dipole anisotropy caused by the superhorizon perturbation is always smaller than the resulting quadrupole as can be seen on the plots.

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* In general case also the amplitudes of *vector* metric perturbations coupled to the spherical harmonics $Y^2_{\pm1}$ and the *tensor* amplitudes times $Y^2_{\pm2}$ would appear in eq.(2.17)

The expansion of the perturbed model is not isotropic and the "Hubble constant" depends on the direction of measurement. Comparing the proper distance to a close object in the direction **n** with its velocity due to the expansion one gets:

$$H(\mathbf{n}) = \frac{\dot{a}}{a} - \frac{1}{2}\dot{h}_{lm}n^l n^m = \frac{\dot{a}}{a}\left[1 - \frac{1}{6}h'(0,x^3)P_0(\mu) + \frac{1}{3}H'(0,x^3)\ P_2(\mu)\right] \tag{2.23}$$

where the variables are calculated at the present time. The monopole part is of no interest. Perturbations in the metric introduce the quadrupole anisotropy to the Hubble law with the relative amplitude:

$$Q_{\rm H} = \frac{1}{3}cos(kx^3)\ H'(0) \tag{2.24}$$

As one can see, the Hubble anisotropy has the same phase of spatial dependence as the quadrupole anisotropy of CMB, but the amplitudes are defined by different quantities.

3. SPHERICAL SOLUTIONS

We follow here the approach to the spherically symmetric world models outsketched in the Appendix B of PP. As in the previous chapter and generally, when the inhomogeneities have the scale much larger than the horizon scale at the epoch of interest, we are going to neglect the influence of pressure gradients on the evolution of the model. We use the Bondi-Tolman metric:

$$ds^2 = c^2dt^2 - X^2(t,r)dr^2 - R^2(t,r)d\Omega^2 \tag{3.1}$$

where r is a radial coordinate and $d\Omega^2$ is the line element on the sphere. (In fact the presence of pressure gradients forces one to use a more general spherically symmetric metric with $g_{tt} = A^2(t,r)$, since the field equations say that the pressure gradient is proportional to the gradient of $A(t,r)$, as shown by May & White (1967). Thus, strictly speaking, the synchronous, comoving coordinate system is impossible in the presence of pressure gradients.) Neglecting pressure gradients from the begining, we adopt line element (3.1) in our calculations.

In the case of a single fluid one can choose the coordinate system (3.1) to be comoving with the matter, so the velocities vanish automatically. In the case of many fluids interacting only gravitationally it is possible that pressure gradients, not necessarilly equal in different components, may cause the relative motion of the fluids. But we are neglecting pressure gradients from the begining, so it is fair to assume that relative motions of the fluids are impossible. Thus it is possible to make the coordinate system comoving with

the matter. In that case the mixed components of the energy-momentum tensor vanish automatically ($T^{tr} \equiv 0$) and, as a consequence of the field equations one has:

$$X = \frac{R_{,r}}{W(r)} \tag{3.2}$$

where $W(r)$ is a free function (Bondi, 1947, PP). Equation (3.2) implies, that one can write down the field equation for $R(t,r)$ in the form:

$$\dot{R}^2 = W^2(r) - 1 + \frac{2Gm(t,r)}{R} \tag{3.3}$$

where

$$m(t,r) = 4\pi \int_0^r \rho R^2 R_{,r} dr = 4\pi \int_0^r \rho R^2 X W dr \tag{3.4}$$

The quantity defined above is the *gravitational* mass in the sphere inside r. The second integral shows that it is the total matter density integrated over the proper volume with some "weighting" function $W(r)$. Equations with the form of Eqs (3.3)and (3.4) are valid in the general case of spherically symmetric configuration with pressure (May & White, 1967). Equation (3.3) has the form of energy equation for a test particle in the field of [possibly variable] mass m. While the density distribution inside the configuration defines the time dependent potential, the function $W(r)$ sets the initial velocity of the particle.

If the pressure vanishes, mass inside any radius r is preserved and Eq (3.3) becomes an ordinary differential equation with coordinate r playing the role of a parameter (PP). In the general case of matter having an admixture of relativistic component (at least photons are such a component) one may solve the evolutionary equation for $R(t,r)$ using the following approach. First we divide the configuration into a number of concentric, thin shells. The innermost spherical region behaves like a part of uniform solution. For this region one has:

$$m^1_{NR}(t) = m^1_{NR}(t_{init}) \qquad m^1_{UR}(t) = m^1_{UR}(t_{init}) \frac{R(t_{init}, r_1)}{R(t, r_1)} \tag{3.5}$$

where m^1_{NR}, m^1_{UR} are the masses of all non-relativistic and ultra-relativistic components, respectively, inside the first zone. The same behavior is true for any other non-relativistic/ultra-relativistic components. During adiabatic expansion a relativistic fluid changes energy density as $\rho_{UR} \sim V^{-4/3}$ where V is the proper volume of the region. Using Eq. (3.4) we have for a thin shell of matter between r_j and r_{j+1}:

$$m^{j+1}_{UR}(t) - m^j_{UR}(t) = \left[m^{j+1}_{UR}(t_{init}) - m^j_{UR}(t_{init})\right] \left[\frac{R^3(t_{init}, r_{j+1}) - R^3(t_{init}, r_j)}{R^3(t, r_{j+1}) - R^3(t, r_j)}\right]^{\frac{1}{3}} \tag{3.6}$$

where we implicitly assumed that $W(r)$ can be treated as constant through a single zone. Using Eq. (3.5) one can obtain solution for $R(t,r_1)$ and $m(t,r_1)$. Knowing $R(t,r_j)$ and

$m(t, r_j)$ one can find $m(t, r_{j+1})$ as a function of $R(t, r_{j+1})$ with the help of Eq. (3.6). Substituting this dependence sinto Eq. (3.5) one finds solution for $R(t, r_{j+1})$. Recurently one can find the metric functions values and their derivatives on a grid. In other points metric can be found by interpolation.

We start calculations early, when ultra-relativistic component of matter dominates the dynamics ($\rho_{UR} >> \rho_{NR}$. At this stage we always set relativistic fluid density to be constant in space. The perturbation is in the free function $W(r)$, which we borrow from PP:

$$W^2(r) = 1 - \frac{r_0^2 - r^2}{r_0^2 + r^2} \frac{r^2}{r_0^2} \tag{3.7}$$

where $r_0 >> 1$ sets the spatial size of perturbation. In our convention coordinate distance $r \approx 1$ corresponds to the present horizon size. The density of non-relativistic component is also set to constant in the case of adiabatic perturbations. For entropy perturbations we use the following shape of initial density of the non-relativistic component:

$$\rho_{NR} \sim \Omega_{NR} \left[2 - \left(\frac{r}{2r_0}\right)^3\right] \qquad r \leq 2r_0 \tag{3.8}$$

Accordingly the specific entropy behaves like:

$$S(r) \sim \rho_{NR}^{-1} \sim \frac{1}{2 - \left(\frac{r}{2r_0}\right)^3} \tag{3.9}$$

We put observers at different places in the world models described above. The co-moving volume is defined as:

$$V(t, r) = \frac{R^2(t, r) X(t, r)}{r^2} \tag{3.10}$$

Observer at any location $(t_{\rm obs}, r)$, where $t_{\rm obs}$ is to represent the epoch of observation, can define the recombination epoch time $t_{\rm rec}$ using the condition:

$$V(t_{\rm obs}, r) = (1 + z_{\rm rec})^3 V(t_{\rm rec}, r) \tag{3.11}$$

where $z_{\rm rec} \approx 10^3$ is the redshift factor of recombination. Suppose we follow a ray, which goes along a null geodesics from the last scaterring surface at $(t_{\rm rec}, r_{\rm rec})$ to the observer at $(t_{\rm obs}, r_{\rm obs})$. The starting point of the photon depends on the direction $\mathbf{n}$ from which it arrives, $r_{\rm rec} = r_{\rm rec}(\mathbf{n})$. The locally measured energies of a photon would be $E_{\rm rec}$ and $E_{\rm obs}$ respectively, the comoving volumes at the locations - $V_{\rm rec}$ and $V_{\rm obs}$. Since initially the radiation temperature was constant through the space and it changes accordingly to

the law $T \sim V^{-1/3}$, we can define it at any time and location. Taking into account the redshift of photons between last scattering surface and the observer we have:

$$T(\mathbf{n}) = T_{\rm local} \ \frac{E_{\rm obs}}{E_{\rm rec}} \ \left(\frac{V_{\rm obs}}{V_{\rm rec}}\right)^{\frac{1}{3}} \tag{3.12}$$

where $T_{\rm local}$ is the locally measured, average temperature of the CMB. We compare results of such CMB temperature measurement in the direction radially in (T_1), radially out (T_3) and in transverse direction (T_2). The average temperature on the sky in our approximation is given as:

$$T_{\rm local} = \frac{1}{4}(T_1 + 2T_2 + T_3) \tag{3.13}$$

We define the dipole and quadrupole anisotropy of the CMB as:

$$D = \frac{T_1 - T_3}{2 \ T_{\rm local}} \qquad Q = \frac{T_1 - 2T_2 + T_3}{4 \ T_{\rm local}} \tag{3.14}$$

According to our calculations the dipole is always about 2 orders of magnitude weaker as compared to the quadrupole. The exception are the places, where the quadrupole vanishes locally, but such places are rare and the a'priori probability of making observations from there is low.

4. DISCUSSION

In sections 2 and 3 we presented a formal demonstration that a perturbation which is initially iso-curvature (i.e. the entropy is perturbed) while the ultra-relativistic component dominates, grows into a curvature (density) perturbation when the overall expansion of the universe makes the non-relativistic component dominant. The dynamics of the universe is believed to be currently dominated by the non-relativistic component (e.g. baryonic, or cold dark matter), while it was dominated in the past by the ultra-relativistic component (e.g. radiation, pairs, etc.). We can envision the expansion history of two universes which differ in the initial entropy per baryon, or any other measure of the ratio of ultra-relativistic to non-relativistic components, both universes being exactly flat, isotropic and homogeneous. *

The two expansion histories are synchronized at $t_0 = 0$. Initially, they are identical, with the scale factor increasing as $t^{1/2}$ while the dynamics is dominated by the ultra-relativistic component. The expansion rate changes to $a \sim t^{2/3}$ at the time $t = t_{eq}$,

* Similar reasoning can be found in Tolman, 1934 and P93.

when the non-relativistic component becomes dominant. This time is different for the two universes with the different ratio of the two components, and therefore, the late expansion rate is different too. This means that the scale factors for the two universes are not the same at any time $t > t_{eq}$. If these two universes are just large parts of the same universe and they come into contact at $t > t_{eq}$, the mis-match of their scale factors will create space curvature, which will give rise to a quadrupole anisotropy of the CMB. On Fig. 2 we show the expansion histories of different parts of our spherical model. At the begining $R(t,r)/r \sim t^{1/2}$ independent of position. In later times the differences become visible.

Our conclusion is that the dipole anisotropy of the CMB cannot be explained with a very large scale entropy gradients in the universe, as proposed by Paczyǹski & Piran (1990). If the dipole is cosmological in origin something else is needed. We see no ‘natural’ explanation. A trivial and entirely ad hoc and artificial ‘explanation’ can be offered. Imagine there are many ultra-relativistic components, with one having a very large scale fluctuation which is exactly in the opposite phase than radiation. There would be no overall change in the ratio of ultra-relativistic to non-relativistic componet, and no effect on the dynamics of the universe, as any effect of the radiation perturbation would be exactly balanced by the opposite effect due to another ultra-relativistic component, yet there would be a dipole in the CMB. There is no justification for such a proposal, and we mention it as an example of what may have to be considered if the Lauer & Postman (1994) result is confirmed and even extended to ever larger scales.

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FIGURE CAPTIONS

Fig.1. The ratio of the dipole D to quadrupole Q anisotropy of the CMB as a function of dimensionless matter density Ω_{NR}. The ratio of the density difference across the horizon Δ to the density perturbation amplitude δ, which is a small number for superhorizon perturbations, serves as a unit of D/Q. (See text for explanations of these parameters). The curves are for $\Omega_{UR} = 10^{-5}$, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1} and 1 from bottom to top. The cosmological constant is given by $\lambda = 1 - \Omega_{NR} - \Omega_{UR}$ in each case and may be negative. (a) results for the *adiabatic* and (b) *isocurvature* perturbations. (See the text for details).

Fig.2. The evolution of expansion factor $R(t, r)/r$ with time for $r = 0.5$, 1.0, 1.5 and $2.0 \times r_0$ (bottom to top). We use the results obtained for our spherical model with $\Omega_{NR} = 0.1$ and $\Omega_{UR} = 10^{-4}$. That illustrates the different expansion rate in different parts of space.